Methodology to Measure Systemic Risk*

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Abstract

Systemic Risk measures reported on CRML website (www.crml.ch) are based on the seminal papers by Acharya et al. (2012) as well as Brownlees and Engle (2017). The paper by Engle et al. (2015) provides an extension handling European particularities.

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1 Capital Requirements

To understand the methodology behind the risk measures, consider an elementary balance sheet of a bank i at time t, with the book value of assets ($BA_{i,t}$) on the left hand side and the book value of equity ($BW_{i,t}$) and debt ($D_{i,t}$) on the right hand side.

Assets ($BA_{i,t}$)	Liabilities
Cash	Equity ($BW_{i,t}$)
Securities	Debt ($D_{i,t}$)
	deposits
Loans	borrowings

One defines the 'quasi-market value' of the assets as the sum of the book value of debt $D_{i,t}$ plus the market value of equity $W_{i,t}$ (also called market capitalization). Formally, $A_{i,t} = D_{i,t} + W_{i,t}$, therefore $A_{i,t} = BA_{i,t} - BW_{i,t} + W_{i,t}$.

What is Systemic Risk? It is the propensity of a firm to be under-capitalized when the financial system as a whole is under-capitalized, i.e., in case of a new financial crisis. A bank is said to be under-capitalized (or in financial stress) if its equity falls below a given fraction θ of its assets, i.e. if $\theta\, A_{i,t} - W_{i,t}$ becomes positive. The parameter θ here is typically a prudential ratio set by the regulator.

Example: Assume $\theta = 8\%$, the assets of the bank are $A_{i,t} = 1000$, and its market capitalization is $W_{i,t} = 80$. Then the bank is just capitalized since 8% of 1000 is 80. Its debt is $D_{i,t} = 920$ and financial leverage is $A_{i,t}/W_{i,t} = 12.5$ (because $12.5 = 1/8\%$). Now assume a new crisis. The financial market breaks down and the credit market dries. The market capitalization of the bank falls well below 80, say 50 (its leverage is now 20). To satisfy the regulatory ratio ($W_{i,t} = 0.08 \times A_{i,t}$), the bank should raise 30 of equity capital from other financial institutions or from the market. However, as the market has dried up, the bank cannot refinance itself and may default, as it happened to Bear Stearns, Lehman Brothers, and others.

In this case, without the massive intervention of central banks and governments, one would probably see a cascade of bank defaults. It is exactly to be prepared for such situations that one needs a measure of the potential level of under-capitalization that a given bank faces.

2 Capital Shortfall

Define $Crisis_{t:t+T}$ an indicator variable that indicates if there is a financial crisis between dates t and $t+T$. The expected capital shortfall of bank i in case of such a crisis is defined as:

$$CS_{i,t:t+T} = E_{t-1}[\theta A_{i,T} - W_{i,T} \mid Crisis_{t:t+T}].$$

It measures how much capital would be needed for that bank as to be again correctly capitalized in case of a new financial crisis. It should be noticed that if the resulting amount of capital is very large, one obtains a link with the notion of *too big to fail*, which remains a source of concern for governments and regulators.

The expected capital shortfall can be rewritten in terms of parameters that can be measured from the balance sheet or estimated econometrically:

$$CS_{i,t:t+T} = \left[\theta(L_{i,t} - 1) - (1-\theta)(1 - LRMES_{i,t:t+T})\right] W_{i,t}.$$

In this expression $L_{i,t} = A_{i,t}/W_{i,t}$ is the *financial leverage* and $LRMES_{i,t:t+T}$ is the *long-run marginal expected shortfall* of the bank, i.e. the sensitivity of its equity return to the evolution of the world market in case of a financial crash. The market capitalization ($W_{i,t}$) and the financial leverage ($L_{i,t}$) are readily available from market and accounting data, respectively. What remains to be estimated is the *LRMES*.

3 Long-Run Marginal Expected Shortfall

LRMES is defined as the sensitivity to a (hypothetical) 40% semiannual market decline:

$$LRMES_{i,t:t+T} = -E_{t-1}\left[R_{i,t:t+T} \mid R_{M,t:t+T} \leq -40\%\right]$$

where $T = 6$ months and cumulative returns are defined as:

$$R_{i,t:t+T} = \exp\left(\sum_{j=1}^{T} r_{i,t+j}\right) - 1$$

and

$$R_{M,t:t+T} = \exp\left(\sum_{j=1}^{T} r_{M,t+j}\right) - 1$$

with $r_{i,t}$ and $r_{M,t}$ the daily log-return of firm i and the daily log-return of the market at date t, respectively.

The measure *LRMES* is particularly difficult to estimate because it corresponds to an extremely rare event. We had only three 40% market crashes over the last century (1929, 2000, and 2008). Brownlees and Engle (2010) advocated for two complementary approaches to estimate the *LRMES*:

- The first (direct) approach consists of estimating the *LRMES* as the expected return of the firm in case of a 40% semiannual decline in the market return. Directly estimating the *LRMES* relies on the simulation of the model over T periods using all information available at date t. It is estimated by:

  $$LRMES_{i,t:t+T} = -\frac{\sum_{s=1}^{S} R_{i,t:t+T}^{(s)} \times \mathcal{I}(R_{M,t:t+T}^{(s)} \leq -40\%)}{\sum_{s=1}^{S} \mathcal{I}(R_{M,t:t+T}^{(s)} \leq -40\%)},$$

  where $\mathcal{I}(x) = 1$ if x is true and 0 otherwise. This approach provides very accurate estimates of the true expectation when the number of simulated data is sufficiently large. In our empirical work, we use $S = 50\ 000$ draws.

- In the second (indirect) approach, the *LRMES* is based on the expected return of the firm in case of a (relatively modest) 2% decline in the daily market return, which is then extrapolated to match a 'once-per-decade" crisis. The sensitivity to a 2% daily world market decline, called Short-run marginal expected shortfall (SRMES) is defined as:

$$SRMES_{i,t} = -E_t\left[R_{i,t+1} \mid R_{M,t+1} \leq -2\%\right].$$

  Then under some (not too straightforward) assumptions, the *LRMES* can be approximated by:

$$LRMES_{i,t:t+T} = 1 - \exp(-k \; SRMES_{i,t}).$$

  The parameter k has been estimated via extreme value theory. For $T = 6$ months, it was found to be close to $k = 18$. This approximation allows a much faster estimation of the risk measures, but does not allow a multifactor approach.

The European risk measures we report are based on the direct approach in a multifactor approach (See Section 5).

The World systemic risk measures we report from the NYU Stern Volatility Lab are based on the indirect approach. A single factor (the World market) is used to estimate the *LRMES*. (The Volatility Lab also reports risk measures based on the direct approach for U.S. financial institutions, assuming a single factor approach.)

4 Systemic Risk of Financial Institutions

We define the systemic risk of bank i as the expected capital shortfall when it is positive:

$$SRISK_{i,t:t+T} = \max(CS_{i,t:t+T}, 0).$$

Having a negative capital shortfall means that the firm has more equity than required by the prudential ratio θ, so that the firm is no at risk.

A nice property of our systemic risk measure is that it allows aggregation. We define the marginal expected shortfall of a given country or the entire financial system as:

$$LRMES_{F,t:t+T} = -E_{t-1}[R_{F,t:t+T} \mid R_{M,t:t+T} \leq -40\%],$$

where $R_{F,t:t+T}$ denotes the cumulative return of the financial industry between t and $t+T$. Because the return of the industry is just the value-weighted sum of the return of the N financial institutions ($R_{F,t:t+T} = \sum_{i=1}^{N} w_{i,t} R_{i,t:t+T}$, with $w_{i,t} = W_{i,t}/W_{F,t}$), we obtain that the marginal contribution of a given institution to the overall *LRMES* is simply the *LRMES* of the institution.

The aggregate expected shortfall can also be obtained by aggregation:

$$LRMES_{F,t:t+T} = \sum_{i=1}^{N} w_{i,t} \; LRMES_{i,t:t+T}.$$

Similarly, the systemic risk of the entire financial system is just:

$$SRISK_{F,t:t+T} = \sum_{i=1}^{N} w_{i,t} \; SRISK_{i,t:t+T}.$$

5 Econometric Methodology

There are substantial differences across European countries in terms of macroeconomic dynamics, fiscal and monetary policy, and regulation. For this reason, we need a fine description of what drives the risk of a financial firm. Our stratification allows for three drivers of a firm's return:

- the country-wide index ($r_{C,t}$),
- the European index ($r_{E,t}$),
- the World index ($r_{W,t}$).

A further complication stems from the asynchronicity of time zones. The stock market of a given country may be affected by a shock on the world index one day later, if the shock is initiated late in the U.S. or overnight in Asia. For these reasons, our system includes five series, $r_t = \{r_{i,t}, r_{C,t}, r_{E,t}, r_{W,t}, r_{W,t-1}\}$.

The objective of the model is to capture the dependence of the return of firm i with respect to the drivers. Our econometric approach aims at capturing this dependence by designing a factor model with time-varying parameters, time-varying volatility, and a general, non-normal dependence structure for the innovations.

We use the following recursive multifactor model with time-varying parameters, after having preliminarily demeaned all return series:

$$\begin{aligned}
r_{i,t} &= \beta_{i,t}^{C}\ r_{C,t} + \beta_{i,t}^{E}\ r_{E,t} + \beta_{i,t}^{W}\ r_{W,t} + \beta_{i,t}^{L}\ r_{W,t-1} + \varepsilon_{i,t} \\
r_{C,t} &= \beta_{C,t}^{E}\ r_{E,t} + \beta_{C,t}^{W}\ r_{W,t} + \beta_{C,t}^{L}\ r_{W,t-1} + \varepsilon_{C,t} \\
r_{E,t} &= \beta_{E,t}^{W}\ r_{W,t} + \beta_{E,t}^{L}\ r_{W,t-1} + \varepsilon_{E,t} \\
r_{W,t} &= \beta_{W,t}^{L}\ r_{W,t-1} + \varepsilon_{W,t},
\end{aligned}$$

where the L superscript corresponds to the lagged world-market index. The parameters of the model are estimated using the Dynamic Conditional Beta approach proposed by Engle (2012). See Engle, Jondeau, and Rockinger (2012) for additional details.

The error terms $\varepsilon_t = \{\varepsilon_{i,t}, \varepsilon_{C,t}, \varepsilon_{E,t}, \varepsilon_{W,t}\}$ are uncorrelated across time and across series, but may be non-linearly dependent both in the time series (such as heteroskedasticity) and in the cross-section (such as tail dependence). To deal with heteroskedasticity, we assume a univariate asymmetric GARCH model:

$$\varepsilon_{k,t} = \sigma_{k,t}\ z_{k,t},$$

where

$$\sigma_{k,t}^2 = \omega_k + \alpha_k \varepsilon_{k,t-1}^2 + \beta_k \sigma_{k,t-1}^2 + \gamma_k \varepsilon_{k,t-1}^2 1_{\{\varepsilon_{k,t-1} \leq 0\}},$$

for $k \in \{i, C, E, W\}$. Innovations $z_t = \{z_{i,t}, z_{C,t}, z_{E,t}, z_{W,t}\}$ are such that $E[z_t] = 0$ and $V[z_t] = I_4$. It is commonly accepted that the conditional distribution of stock market returns is fat-tailed and asymmetric. To capture these features, the innovations are assumed to have a univariate skewed t distribution, $z_{k,t} \sim f(z_{k,t}; \nu_k, \lambda_k)$, where f denotes the pdf of the skewed t distribution, with ν_k the degree of freedom and λ_k the asymmetry parameter.

Although the innovations z_t have been preliminarily orthogonalized, they are not a priori independent. Their joint distribution should allow for possible non-linear dependencies.

A convenient modeling approach is to use *copula*. We define $u_t = \{u_{i,t}, u_{C,t}, u_{E,t}, u_{W,t}\}$ as the margin of z_t with $u_{k,t} = F(z_{k,t}; \nu_k, \lambda_k)$, where F is the cdf of the skewed t distribution with parameters (ν_k, λ_k). The copula is then the joint distribution of u_t, denoted by $C(u_t)$.

After investigating several alternative copulas, we selected the t copula, which has been found to capture the dependence structure of the data very well. It accommodates tail dependence and its elliptical structure provides a convenient way to deal with large-dimensional systems. The t copula is defined as:

$$C_{\Gamma,\bar{\nu}}(u_{i,t}, ..., u_{W,t}) = t_{\Gamma,\bar{\nu}}(t_{\bar{\nu}}^{-1}(u_{i,t}), ..., t_{\bar{\nu}}^{-1}(u_{W,t})),$$

where $t_{\bar{\nu}}$ is the cdf of the univariate t distribution with degree of freedom $\bar{\nu}$ and $t_{\Gamma,\bar{\nu}}$ is the cdf of the multivariate t distribution with correlation matrix Γ and degree of freedom $\bar{\nu}$.

To summarize, our model combines a Dynamic Conditional Beta model for the returns' dynamic, univariate GARCH models for the dynamic of the volatility of the error terms, and a t copula for the dependence structure between the innovations. To deal with the possible time variability of some of the model parameters, we estimate the model over a rolling window of 10 years of data as soon as a new observation is made available.

6 A Comment on Units

All data are expressed in billion euros.

One key ingredient of systemic risk measures is firm's financial leverage. An important issue in measuring leverage is that the firms in the U.S. and Europe are currently under two different accounting standards: Generally Accepted Accounting Principles (GAAP) in the U.S. and International Financial Reporting Standards (IFRS) in Europe. The balance sheet of U.S. banks presents derivatives on a net basis, meaning that derivatives represent a small part of the assets, whereas the balance sheet of European banks reports derivatives on a gross basis. Some crude estimates suggest that the total assets (and therefore the leverage) of large U.S. banks (which are highly active in derivatives markets) would be 40-60% larger under IFRS than under U.S. GAAP. To deal with this important source of bias, we use $\theta = 5.5\%$ for Europe institutions.

7 Update of the methodology for Cooperative Banks

As of end of January 2014, risk measures have been adjusted for two large French co-operative banking groups, namely Crédit Agricole and BPCE. More precisely, we have moved from measures for the bank (Crédit Agricole SA and Natixis) to measures for the group (Crédit Agricole Group and BPCE Group). This change is the conclusion of a long, internal as well as external, debate, on the way to deal with co-operative or mutual capital structures. In the case of Crédit Agricole and BPCE, we had to face two issues: (1) Both groups are among the largest banks in Europe with a high leverage, so that they have significant contributions to the aggregate European SRISK measure. (2) The capital structure of both groups is relatively complex and this complexity renders difficult the computation of their actual risks and leverage.

We briefly describe the capital structure of Crédit Agricole Group and BPCE Group, the way we have adjusted the methodology, and the risk measures as of end of June 2013.

7.1 Capital structure of Crédit Agricole

Crédit Agricole Group (CAG) is composed of two types of entities:

- The bank (Crédit Agricole SA, CASA) is involved in global financial activities, investment banking, international operations, etc. It includes Crédit Agricole CIB (investment banking division), Amundi (its asset management entity), and LCL (a large French bank). Crédit Agricole SA is the main listed entity of the group.
- The Regional banks (RBs) are co-operative entities, involved in retail banking, mostly lending to their clients, firms and households. Some RBs (13 over 39) are listed, but the floating is very limited and certificates are highly illiquid.

The 39 RBs jointly hold (via SAS Rue la Boétie) 56.3% of CASA. CASA itself holds approximately 25% stakes in most of the RBs (except the Corsican regional bank).

The balance sheet of Crédit Agricole Group consolidates the balance sheets of CASA and RBs. The main issue concerns the entity that will guaranty the liquidity and solvency of CASA if needed.

As stated in the "Update A01 of the 2012 Registration document" of Crédit Agricole Group (page 29), "it is important to recall that the Regional Banks have granted Crédit Agricole S.A. their joint and several guarantee up to the total of their capital and reserves (€54 billion at 31 December 2012), should Crédit Agricole S.A. prove unable to meet its obligations. This guarantee reciprocates the commitment of Crédit Agricole S.A., as central body, to maintain the solvency and the liquidity of the Regional Banks. Consequently, international rating agencies award the same ratings to the issuance programmes of Crédit Agricole S.A. and of the rated Regional Banks." Consequently, given the full commitment of the various entities with respect to each other, CAG is considered by the French and European authorities as the scope of consolidation for Crédit Agricole. Also rating agencies attribute the same ratings to the issuance programs of CASA and of the rated RBs.

7.2 Capital structure of BPCE Group

BPCE Group is a cooperative banking group formed in 2009 from the merger of two cooperative banks, Groupe Caisses d'Epargne and Groupe Banque Populaire. It is composed of three types of entities:

- The bank (Natixis) is involved in global financial activities, investment banking, international operations, etc. Natixis is the main listed entity of the group.
- BPCE (for Banque Populaire Caisses d'Epargne) is the central institution of the Group. It holds 71.98% of Natixis. It has other subsidiaries such as Crédit Foncier or Crédit Maritime Mutuel.
- 19 Banques Populaires and 17 Caisses d'Epargne are co-operative entities, involved in retail banking, mostly lending to their clients, firms and households. The Banques Populaires hold 50% of BPCE and the Caisses d'Epargne hold 50% of BPCE. Banques Populaires and Caisses d'Epargne are held by 8.7 million cooperative shareholders and are not listed.

As stated in "Affiliation of Natixis to BPCE and guaranty and solidarity system within Groupe BPCE" (page 1), BPCE "must take all necessary measures to guaranty the liquidity and solvency of Groupe BPCE and institutions affiliated with BPCE as central body, as well as to organize the financial solidarity within Banque Populaire and Caisse d'Epargne networks. [...] Should the situation of Natixis require the triggering of the guaranty and solidarity system, BPCE may draw financial means from four different and complementary sources: firstly BPCE will draw on its own capital (in compliance with its shareholder duties); secondly it will call upon the Mutual Guaranty Fund; thirdly it will make a call on the two networks' guaranty funds (Banques Populaires and Caisses d'Epargne); finally BPCE will request the contribution capacity of the Banques Populaires and the Caisses d'Epargne (37 credit institutions) up to the full amount of their equity."

7.3 Methodology for cooperative or mutual banks

We investigated different approaches to adjust our methodology to the case of mutual banks, where several entities may in fact constitute a unique group, with not all entities being listed on financial markets. For instance, for Credit Agricole, aggregating the equity of the different entities would be a good solution. However, only a limited number of RBs are listed, with in addition very limited liquidity (floating equity is approximately 1.5% of total market capitalization of the listed RBs). For BPCE, the cooperative banks are not listed on financial markets. As a consequence, no objective market-based value of equity can be found.

Our approach therefore consists in applying to the group the adjusted book-to-market (BTM) ratio of the bank to measure the "implicit" market capitalization of the group. The adjusted BTM ratio is defined as:

$$BTM_{bank} = \frac{\text{book equity}_{bank} - \text{intangibles}_{bank}}{\text{market capitalization}_{bank}} \tag{1}$$

We exclude goodwill and other intangibles, following the approach used for computing Tier 1 Equity Capital, as they cannot be used to offset losses.

Then, we define the "implicit" market capitalization of the Group as:

$$\text{market capitalization}_{group} = \frac{\text{book equity}_{group} - \text{intangibles}_{group}}{BTM_{bank}} \tag{2}$$

References

Acharya VV, Pedersen LH, Philippon T, Richardson MP. 2012. Measuring systemic risk. CEPR Discussion Paper no. 8824.

Brownlees CT, Engle RF. 2017. SRISK: A conditional capital shortfall measure of systemic risk. *Review of Financial Studies* **30(1)**: 48–79.

Engle R, Jondeau E, Rockinger M. 2015. Systemic risk in europe. *Review of Finance* **19**: 145–190.